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**OPTIMAL RESOURCE ROYALTIES WITH UNKNOWN
AND TEMPORALLY INDEPENDENT EXTRACTION
COST STRUCTURES***

BY GÉRARD GAUDET, PIERRE LASSERRE, AND NGO VAN LONG¹

We study optimal nonrenewable resource royalty contracts when the extracting agent has private information on costs. This is a dynamic incentive problem in which the repeated relationship between the principal and the agent is constrained by initial reserves. Commitment is limited to one period and costs are intertemporally independent. Compared with full information extraction, information asymmetry shifts production to the future when the optimal contract requires exhaustion in two periods. When exhaustion by all types in two periods is not warranted, the effect on the terminal period is ambiguous and the output of even the lowest cost firm is always distorted.

1. INTRODUCTION

In nonrenewable natural resource exploitation, one often observes that the owner of the rights to a stock of the resource (a mine) will delegate the extraction of the resource (the mining operation) to a firm specialized in resource extraction, in return for preestablished royalty payments. For example, in many countries surface rights are distinct from mineral rights and the government retains the mineral rights even though land is privately owned. The exclusive right to search for and extract minerals then typically takes the form of a lease over the life of the mine to a private firm or individual.² There are also many instances where private holders of the mineral rights will entrust the actual extraction process to some specialized agent. For example, the mining prospector who holds the rights to a discovery will often enter into a contract with a mining firm which will carry out the exploitation of the mine. The question then arises: What is the optimal royalty contract from the point of view of the mine owner?

If the resource price and the extraction costs are perfectly observable by both the mine owner and the mine manager, then the answer is straightforward. The well known Hotelling rule of natural resource extraction (Hotelling 1931) implies that if the mine owner wishes to maximize the present value of net benefits over the life of the mine, the royalty schedule must induce the mining firm to exhaust the mine in such a way that marginal net benefits grow over time at the rate of discount.

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² The major part of the royalty payment for exclusive use of the mineral rights often takes the form of a mining tax, which applies specifically to all mining firms in the particular mining industry.

In practice, however, only the mining firm will fully know the true costs of extraction. This asymmetry of information creates a situation where adverse selection may occur and the optimal royalty must then take into account some informational constraints. How will the optimal royalty under adverse selection compare to that which would prevail in the full information context? How will it vary over time? What distortion must be introduced in the Hotelling rule in order to discourage the mining firms from exaggerating their costs? What is the direction of intertemporal bias resulting from adverse selection? These are the types of issues addressed in this paper.

The problem constitutes an application of the well known principal-agent problem with adverse selection, already extensively studied and applied to various situations with asymmetry of information.³ For the sake of the discussion, we will throughout the paper think of the principal as being the government.⁴ We will assume that although the government commits itself to the current period's royalty rule, it cannot commit itself to future royalty rules.⁵ The impossibility of intertemporal commitment seems like a natural assumption to make, since generally the extent to which a government can bind future ones in such a matter is very limited.⁶ The problem raised is inherently dynamic in nature and hence must be posed as a multi-period principal-agent problem. There are actually two potential sources of dynamics. The first is due to the intertemporal link arising from the fact that future choices are physically constrained by current decisions. Thus current net benefits must continually be weighted against future net benefits when deciding on current actions, even in a world of symmetric information. In the present case, this link is captured by the cumulative capacity constraint on the agent's action attributable to the nonrenewability of the resource stock. The second potential source of dynamics is specific to the information problem. It will arise when the true value of the parameter which is subject to asymmetry of information is temporally correlated. The principal can then infer from the agent's current action some information about

³ See for example Baron and Myerson (1982), Guesnerie and Laffont (1984), Laffont and Tirole (1986). Useful surveys of various aspects of mechanism design with incomplete information are contained in Baron (1989), Besanko and Sappington (1987) and Caillaud, Guesnerie, Rey, and Tirole (1988).

⁴ Notice that unlike in the traditional regulation problem, the government's objective here is not to induce efficient production decisions from a *monopolist* with limited cost information, as for instance in Baron and Myerson (1982), Besanko and Sappington (1987) or, in the case of a nonrenewable resource, Karp and Livernois (1992). Rather, the principal's objective in this paper is to capture the resource rent generated by price-taking resource extractors under asymmetry of information on costs.

⁵ We therefore seek a closed-loop solution. An open-loop solution is appropriate in the case where the government can commit intertemporally. This case is analysed in Gaudet, Lasserre, and Long (1991). It is shown there that the possibility of intertemporal commitment will generally lead to a different output path, although there exists an interesting special case where it will not matter.

⁶ See Laffont and Tirole (1993, chapter 9) for some other examples of lack of long-term commitment based on this reason. The impossibility to commit in a relationship between private parties will arise in situations where complete contracts—contracts that take into account all future contingencies—are either too costly or simply impossible to write. See Grossman and Hart (1986), Laffont and Tirole (1988) as well as Laffont and Tirole (1993).

the value of the future parameters the agent will be facing. Thus, in the present case, if the cost parameters were temporally correlated an inter-period link would arise from the fact that the report made by the firm in one period provides information to the government about the true value of its cost parameters in future periods.

The consequences of this second source of dynamics in the absence of pure long-term commitment are well known.⁷ One of those consequences is to give rise to a ratchet effect, whereby a low-cost firm has an incentive not to perform so as to reveal its true type, since the principal would then be more demanding in the next period in order to capture the firm's rent. As a result, the equilibria will generally involve pooling of types and the revelation principle—by which one can restrict attention to incentive compatible direct mechanisms—breaks down. We will in this paper intentionally ignore this source of dynamics by assuming that the true value of the cost parameter which is subject to asymmetry of information is not correlated over time. Thus the principal can learn nothing about the agent's future type from being informed of its current type. By concentrating on this polar case we are able to better focus on the less well known qualitative implications for optimal incentive mechanisms under adverse selection of the first source of dynamics, namely that generated by the nonrenewability constraint.⁸ Thus we will be able to highlight some effects of asymmetric information which cannot be captured either in a purely static framework or in a more simple repeated relationship, whether with temporally correlated costs or not. The zero correlation assumption also restores the revelation principle,⁹ of which we will make ample use.

In Section 2 we present the basic model which is used to analyse this problem. In Section 3 we solve for the reference scenario of symmetric information. The extraction path will in that case satisfy the usual Hotelling rule, with its characteristic declining rate of extraction when the rate of discount is positive. Section 4 is devoted to characterizing the optimal incentive mechanism for the case where the agent has private information on extraction costs. We show that the need to provide the mining firm an incentive not to exaggerate its extraction costs will result in an equilibrium extraction path which, generally speaking, favors the first period less than under symmetric information. It may in fact require an increasing rate of extraction. This effect may be reversed however if the parameters are such that the optimal royalty requires of some of the firms that they leave part of their resource stock unexploited. In that case even the firm with the lowest cost will be imposed a

⁷ See for instance Baron and Besanko (1984), Baron (1989) or Laffont and Tirole (1988, 1993).

⁸ There is admittedly some loss in realism in assuming that the types are not at all correlated over time, but there would also be in assuming that the types are perfectly correlated over time. For instance knowing the current grade of ore in a mine, which could be a determinant of its current cost, might provide only very limited information on the grades and hence the costs that will be met in some future periods, as the mine gets depleted. We believe that whatever loss in realism there is in the present context is compensated by some gain in insight.

⁹ The proof of this is a simple adaptation of the proof of the revelation principle in the one-period framework, as given in for instance Laffont and Tirole (1993).

distortion in its extraction path via the optimal royalty scheme, in contrast with the usual static optimal incentive mechanisms.

For the purpose of Sections 2 to 4, it is sufficient to limit attention to a situation where the optimal terminal period never exceeds the second period. But the asymmetry in information may also have an effect on the optimal terminal period. To properly study this we have to introduce the possibility of exploiting optimally beyond the second period. This is done in Section 5 and in the Appendix. We show that the effect of asymmetry of information on the terminal period is ambiguous. Allowing for an arbitrary number of periods also provides further insight into the reason for the above mentioned distortion imposed on the lowest-cost firm and shows this to be in fact a robust result in this dynamic setting. The reason for this result is that the opportunity cost of depleting the stock of resource in the current period rather than leaving it for future use is generally lower under asymmetric information than under symmetric information. This is due to the distortions required in the future periods because of the information constraints. Thus even though no distortion to the lowest-cost firm is necessary in the current period on account of current period asymmetry of information, it will still be required to increase its current production in order to reduce the amount of future information inefficiencies. It thus appears that the classic result in the theory of incentives that the output of the lowest-cost firm always remains efficient under asymmetric information is specific to the static framework.

In Section 6, we discuss the royalty payment scheme which corresponds to the optimal revelation mechanism. Taking the two periods case as an example, it is shown, in particular, how the government will extract second period expected information rents using a lump sum payment in the first period. We end with a brief conclusion in Section 7.

2. THE MODEL

To model the problem of determining the optimal royalty when the mining firm's cost structure is not known to the government, we assume that the firm's costs at period t are given by some function $C(q_t, \theta_t)$, where q_t is the quantity of the resource extracted in period t and θ_t is a cost parameter known only to the mining firm. The parameter θ_t can reflect various aspects of the firm's efficiency, as is usual in such models. We however restrict attention in this paper to the case where the θ_t s are not correlated over time. Thus each firm draws a new independent θ_t at each t .

The government can observe output (the quantity extracted) but it cannot verify the cost actually incurred by the firm. Therefore it cannot base its royalty schedule on the true extraction costs of the firm. For, since its true costs cannot be observed by the government, we must expect that the firm would, if asked for a cost report, lie about its true cost function whenever it is advantageous to do so.

To keep things tractable, we will take the cost function to be of the form

$$(1) \quad C(q_t, \theta_t) = \theta_t q_t + \frac{b}{2} q_t^2, \quad b \geq 0.$$

Hence marginal cost is¹⁰

$$(2) \quad MC(q_t, \theta_t) = \frac{\partial C(q_t, \theta_t)}{\partial q_t} = \theta_t + bq_t.$$

The firm knows both θ_t and b . The government, however knows only b and the cumulative distribution function of θ_t , $F(\theta_t)$, defined on $[\theta^L, \theta^H]$. To this distribution function is associated the density function $f(\theta_t) > 0$, assumed differentiable on $[\theta^L, \theta^H]$. Knowledge of this probability distribution is shared by both the government and the firm.

We assume for simplicity that the resource becomes worthless after the second period. It is therefore never optimal to exploit the resource beyond period 2 and we can limit the analysis to two periods. We relax this assumption in Section 5 and in the Appendix, where we discuss the effect of asymmetry in information on the optimal terminal date.¹¹

The government wants to set a royalty schedule $R_t = R_t(q_t)$, $t = 1, 2$, that maximizes expected social welfare. Social welfare is taken to be a weighted sum of consumers' surplus, the government's revenue and producers' surplus. To simplify the problem, we assume that the country is a price taker in the world market, so that domestic production of the resource good does not give rise to consumers' surplus. Thus we may write social welfare as

$$(3) \quad W_t = R_t + \alpha \Pi_t$$

where

$$(4) \quad \Pi_t = p_t q_t - C(q_t, \theta_t) - R_t(q_t)$$

is producers' surplus. The exogenously given world price, p_t , is assumed known to both the government and the firm.

We adopt the standard assumption that $0 < \alpha < 1$: a dollar in government revenue is valued more highly than a dollar that remains as profits in the hands of the firm. For if $\alpha \geq 1$, maximum social welfare is attained when the firms' profit is maximized. Hence the incentive problem becomes trivial: the government should set the royalty payments to zero and leave the decisionmaking to the firm, knowing that it will use all its private information in order to maximize profit.

¹⁰ The fact that total and marginal costs move in the same direction when θ_t changes guarantees that the "single crossing property" is satisfied.

¹¹ As is frequently done in dynamic programming problems, when the price of the resource is positive beyond the second period our solution can be worked out backward to characterize the whole equilibrium path for a T -period problem. The method is sketched in the Appendix. If the resource is worthless after the second period, clearly $T \leq 2$.

We model the problem as a direct revelation game. Hence the government chooses an incentive mechanism, in the form of a pair $(R_t(\tilde{\theta}_t), q_t(\tilde{\theta}_t))$, that is optimal given the optimal response $\tilde{\theta}_t$ of the firm, where $\tilde{\theta}_t$ denotes the value of its cost parameter as reported by the firm. Given that mechanism, the firm then chooses its optimal response, in the form of a $\tilde{\theta}_t$, the value of which will depend on θ_t , the true value of its parameter. According to the revelation principle,¹² we can restrict attention to mechanisms in response to which the firm will find it optimal to reveal the true value of its cost parameter—mechanisms such that $\tilde{\theta}_t = \theta_t$ —and, knowing $q_t(\theta_t)$ and $R_t(\theta_t)$, we can obtain $R_t(q_t)$ by inverting $\theta_t = \theta_t(q_t)$.

In order to be implementable, an incentive scheme must also leave the firm sufficient surplus to cover its opportunity costs. Otherwise it would rationally choose not to participate. We assume that the firm may opt out at the beginning of each period, in response to the royalty scheme proposed by the government.

In choosing its optimal royalty the government is also constrained by the nonrenewability of the resource. Hence if X_t is the stock of the resource in the ground at the beginning of period t , then $q_1 \leq X_1$ and $q_2 \leq X_2 = X_1 - q_1$.

The fact that the government cannot commit itself in period 1 about the royalty schedule of period 2 implies that we must seek a closed-loop solution to the government's problem. In other words, we must determine the optimal royalty schedule for each period as a function of the beginning of period stock of the resource. We do this by solving backwards for the government's problem, starting with period 2 (the final period).

3. THE SYMMETRIC INFORMATION CASE

Before going on to solve for the optimal royalty scheme under asymmetry of information, we first derive the properties of the royalty schedule which would maximize social welfare in the case where the government shared the firm's information about its cost structure. This symmetric information scenario is a useful benchmark, since it yields a first-best solution.

To do this, assume that at the beginning of period t (before the decision on q_t is made), the exact value of θ_t is revealed both to the firm and to the government. In the last period, the government then wishes to maximize

$$(5) \quad W_2 = R_2 + \alpha \Pi_2$$

subject to

$$(6) \quad 0 \leq q_2 \leq X_2$$

and $\Pi_2 \geq 0$, where $\Pi_2 = p_2 q_2 - C(q_2, \theta_2) - R_2$. Clearly, since $0 \leq \alpha < 1$, the solution consists of choosing q_2 to maximize $p_2 q_2 - C(q_2, \theta_2)$ subject to (6), and then

¹² See Baron and Myerson (1982), Guesnerie and Laffont (1984) or Baron (1989).

set R_2 so as to collect that maximized value as royalties. Any larger royalty would of course induce the firm to withdraw its participation in the second period.

For an interior solution, a necessary condition is $p_2 - MC(q_2(\theta_2), \theta_2) = 0$ or, in terms of the cost function (1),

$$(7) \quad p_2 - \theta_2 - bq_2 = 0.$$

Let $q_2^s(\theta_2, X_2)$ denote the optimal resource extraction in period 2 from the government's standpoint when taking into account the constraints $0 \leq q_2(\theta_2) \leq X_2$. If we define z_2^s and y_2^s respectively by $z_2^s = p_2 - bX_2$ and $y_2^s = p_2$ and set $\underline{\theta}_2^s = \min[z_2^s, \theta^H]$ and $\bar{\theta}_2^s = \max[y_2^s, \theta^L]$, we may write $q_2^s(\theta_2, X_2)$ as¹³

$$(8) \quad q_2^s(\theta_2, X_2) = \begin{cases} X_2 & \text{if } \theta^L \leq \theta_2 \leq \underline{\theta}_2^s \\ \frac{1}{b}[p_2 - \theta_2] & \text{if } \underline{\theta}_2^s < \theta_2 < \bar{\theta}_2^s \\ 0 & \text{if } \bar{\theta}_2^s \leq \theta_2 \leq \theta^H. \end{cases}$$

Note that $\underline{\theta}_2^s < \theta^L$ or $\bar{\theta}_2^s > \theta^H$ may occur. The sets $\{\theta_2 | \theta_2 \in [\theta^L, \underline{\theta}_2^s]\}$ or $\{\theta_2 | \theta_2 \in [\bar{\theta}_2^s, \theta^H]\}$, respectively, would then be empty. Note also that the value of $\underline{\theta}_2^s$ depends on X_2 , which itself depends on θ_1 . Thus X_2 is endogenous, unlike X_1 .

Now denote by $\Gamma^s(X_2)$ the expected royalty for period 2 under symmetric information. Since the optimal choice of R_2 for any q_2 from the government's point of view is $R_2 = p_2 q_2 - C(q_2, \theta_2)$, we know that $\Pi_2 = 0$. Therefore the expected social welfare for period 2 is

$$(9) \quad \begin{aligned} EW_2 = \Gamma^s(X_2) = & \int_{\theta^L}^{\underline{\theta}_2^s} [p_2 X_2 - C(X_2, \theta_2)] f(\theta_2) d\theta_2 \\ & + \int_{\underline{\theta}_2^s}^{\bar{\theta}_2^s} [p_2 \tilde{q}_2^s(\theta_2) - C(\tilde{q}_2^s(\theta_2), \theta_2)] f(\theta_2) d\theta_2 \end{aligned}$$

where $\tilde{q}_2^s(\theta_2) \equiv [p_2 - \theta_2]/b$, the solution to (7). From (9) we derive

$$(10) \quad \frac{d\Gamma^s}{dX_2} = \int_{\theta^L}^{\underline{\theta}_2^s} (p_2 - \theta_2 - bX_2) f(\theta_2) d\theta_2.$$

We will assume, to simplify, that $p_2 \geq \theta^H + bX_1$. This assumption guarantees that under symmetric information, all firm types will be required to exhaust the initial stock of the resource by the end of period 2.¹⁴ We then necessarily have $\underline{\theta}_2^s = \theta^H$

¹³ The superscript s refers to the symmetric information situation. We will later use the superscripts a to denote the solution under asymmetry of information. It will become clear in Sections 4 and 5 why it is useful to introduce here the notation z_2^s and y_2^s .

¹⁴ This assumption is easily relaxed. Doing so would simply increase the number of cases to be dealt with further on, with no gain in insight.

and hence (10) becomes

$$(11) \quad \frac{d\Gamma^s}{dX_2} = p_2 - E\theta_2 - bX_2 = p_2 - EMC(X_2, \theta_2).$$

Since $X_2 = X_1 - q_1$, the first-period problem for the government is to maximize

$$(12) \quad V = R_1 + \alpha [p_1 q_1 - C(q_1, \theta_1) - R_1] + \delta \Gamma^s(X_1 - q_1)$$

subject to $0 \leq q_1 \leq X_1$ and to $\pi_1 \geq 0$, where $\delta < 1$ is the discount factor. Again with $0 \leq \alpha < 1$, the optimal choice of R_1 for any q_1 must be $R_1 = p_1 q_1 - C(q_1, \theta_1)$. Therefore the optimal solution consists in finding for each $\theta \in [\theta^L, \theta^H]$ the value of q_1 (call it $q_1^s(\theta_1, X_1)$) which maximizes

$$(13) \quad V = p_1 q_1 - C(q_1, \theta_1) + \delta \Gamma^s(X_1 - q_1)$$

subject to $0 \leq q_1 \leq X_1$, and in setting the first-period royalty so as to capture all the resulting rent, thus just respecting the firm's participation constraint.

For an interior solution ($0 < q_1 < X_1$), a necessary condition is

$$(14) \quad p_1 - \theta_1 - bq_1 = \delta \frac{d\Gamma^s(X_1 - q_1)}{dX_2}.$$

If we denote the solution to (14) by $\tilde{q}_1^s(\theta_1)$, it follows that

$$(15) \quad q_1^s(\theta_1, X_1) = \begin{cases} X_1 & \text{if } \theta^L \leq \theta_1 \leq \underline{\theta}_1^s \\ \tilde{q}_1^s(\theta_1) & \text{if } \underline{\theta}_1^s < \theta_1 < \bar{\theta}_1^s \\ 0 & \text{if } \bar{\theta}_1^s \leq \theta_1 \leq \theta^H \end{cases}$$

where $\underline{\theta}_1^s = \min[z_1^s, \theta^H]$ and $\bar{\theta}_1^s = \max[y_1^s, \theta^L]$, with z_1^s and y_1^s defined by

$$z_1^s = p_1 - bX_1 - \delta \frac{d\Gamma^s(0)}{dX_2}$$

and by

$$y_1^s = p_1 - \delta \frac{d\Gamma^s(X_1)}{dX_2}.$$

Note that since the resource is exhausted in period 2 for all θ_2 ,

$$(16) \quad \bar{q}_1^s(\theta_1) = \frac{p_1 - \theta_1 - \delta[p_2 - E\theta_2 - bX_1]}{(1 + \delta)b}.$$

For $\underline{\theta}_1^s < \theta_1 < \bar{\theta}_1^s$, resulting in an interior solution for q_1 , condition (14) can be written, after substitution for $d\Gamma^s(X_1 - q_1)/dX_2$ from (11),

$$(17) \quad p_1 - MC(q_1, \theta_1) = \delta[p_2 - EMC(X_1 - q_1, \theta_2)].$$

Hence the royalty schedule is chosen so that (expected) marginal profits grow at the rate of discount. This is the usual Hotelling rule.

Since θ_t can be observed by the government at the beginning of each period, a royalty rule which will realize the optimal extraction program, from the government's point of view, can be specified as follows:

$$(18) \quad R_t = \begin{cases} p_t q_t^s(\theta_t, X_t) - C(q_t^s(\theta_t, X_t), \theta_t) & \text{if } q_t = q_t^s(\theta_t, X_t) \\ p_t q_t - C(q_t, \theta_t) + k, k > 0 & \text{if } q_t \neq q_t^s(\theta_t, X_t) \end{cases}$$

for $t = 1, 2$ and where $X_2 = X_1 - q_1^s(\theta_1, X_1)$. This rule will ensure that the firm chooses $q_t^s(\theta_t, X_t)$ at each t , since any other choice of output leaves it with a negative surplus. Total discounted resource rent will be maximized and captured entirely by the government.

4. THE ASYMMETRIC INFORMATION PROBLEM

Consider now the situation where the true value of θ_t is known only to the firm. As already mentioned, we make use of the revelation principle in solving the problem in the presence of asymmetry of information. We may therefore limit our attention to the class of incentive compatible mechanisms. We show in the Appendix how this class of mechanisms can be characterized, taking into account the non-renewability constraint.

4.1. The Second-Period Solution. Consider the last period first. The government asks the firm that reports $\tilde{\theta}_2$ in period 2 to produce $q_2(\tilde{\theta}_2)$ in that period and to pay the government the total royalty $R_2(\tilde{\theta}_2)$. The surplus of the firm in period 2 if it reports $\tilde{\theta}_2$ when the true cost parameter takes the value θ_2 will therefore be

$$p_2 q_2(\tilde{\theta}_2) - \theta_2 q_2(\tilde{\theta}_2) - \frac{b}{2} q_2(\tilde{\theta}_2)^2 - R_2(\tilde{\theta}_2).$$

It should be borne in mind that $q_2(\tilde{\theta}_2)$ and $R_2(\tilde{\theta}_2)$, and therefore the firm's eventual surplus in period 2, will depend on X_2 as well as on θ_2 . It has been suppressed temporarily as an argument only to alleviate the notation.

If we denote the firm's surplus by $\phi(\tilde{\theta}_2; \theta_2)$ and let $\Pi_2(\theta_2) \equiv \phi(\theta_2; \theta_2)$ for a given X_2 , we can write (see the Appendix) the government's objective function as

$$(19) \quad \begin{aligned} EW_2 = \int_{\theta^L}^{\theta^H} & \left[p_2 q_2(\theta_2) - \theta_2 q_2(\theta_2) - \frac{b}{2} q_2(\theta_2)^2 \right. \\ & \left. - (1 - \alpha) q_2(\theta_2) h(\theta_2) \right] f(\theta_2) d\theta_2 \\ & - (1 - \alpha) \Pi_2(\theta^H) \end{aligned}$$

where

$$h(\theta_t) = \frac{F(\theta_t)}{f(\theta_t)}, \quad t = 1, 2.$$

The government's problem can now be stated as that of choosing $q_2(\theta_2)$, subject to $0 \leq q_2(\theta_2) \leq X_2$, and $\Pi_2(\theta^H)$, subject to $\Pi_2(\theta^H) \geq 0$, in order to maximize (19). Having determined the solution for $q_2(\theta_2)$ and verified that it satisfies incentive compatibility constraint (A.5), the optimal royalty for period 2 can be found from (A.4).

Clearly, the solution requires that we set $\Pi_2(\theta^H) = 0$: highest cost mines must yield zero period 2 surplus under an optimal incentive scheme. As for $q_2(\theta_2)$, an interior solution must satisfy

$$(20) \quad p_2 - \theta_2 - bq_2 - (1 - \alpha)h(\theta_2) = 0.$$

Now define the extended function $g(\theta_t)$, $t = 1, 2$ as

$$(21) \quad g(\theta_t) \equiv \begin{cases} h(\theta^L) & \text{if } \theta_t < \theta^L \\ h(\theta_t) & \text{if } \theta^L \leq \theta_t \leq \theta^H \\ h(\theta^H) & \text{if } \theta^H < \theta_t \end{cases}$$

and let z_2^a be given by

$$(22) \quad p_2 - z_2^a - (1 - \alpha)g(z_2^a) = bX_2$$

and y_2^a be given by

$$(23) \quad p_2 - y_2^a - (1 - \alpha)g(y_2^a) = 0.$$

If we set $\underline{\theta}_2^a = \min[z_2^a, \theta^H]$ and $\bar{\theta}_2^a = \max[y_2^a, \theta^L]$, then, taking into account the

constraints $0 \leq q_2 \leq X_2$, the full solution for q_2 can be written¹⁵

$$(24) \quad q_2^a(\theta_2, X_2) = \begin{cases} X_2 & \text{if } \theta^L \leq \theta_2 \leq \underline{\theta}_2^a \\ \frac{1}{b} [p_2 - \theta_2 - (1 - \alpha)h(\theta_2)] & \text{if } \underline{\theta}_2^a < \theta_2 < \bar{\theta}_2^a \\ 0 & \text{if } \bar{\theta}_2^a \leq \theta_2 \leq \theta^H. \end{cases}$$

Notice that since $\underline{\theta}_2^a \neq z_2^a$ whenever $z_2^a > \theta^H$ and $\bar{\theta}_2^a \neq y_2^a$ whenever $y_2^a < \theta^L$, the arbitrariness of the extended function $g(\theta_2)$ is of no consequence for the results. It simply facilitates the definition of the threshold levels of θ_2 by overcoming the fact that the function $h(\theta_i)$ itself is defined only for $\theta_i \in [\theta^L, \theta^H]$.

We follow the incentive literature in assuming that

$$(25) \quad \frac{dh(\theta_i)}{d\theta_i} \geq 0 \quad i = 1, 2.$$

This is the so-called "monotone hazard rate" assumption. It is sufficient to insure that $q_2^a(\theta_2, X_2)$ is a nondecreasing function of θ_2 and therefore satisfies condition (A.5) for incentive compatibility.

The expected surplus of a firm in period 2 is

$$(26) \quad \Psi(X_2) \equiv \int_{\theta^L}^{\theta^H} \Pi_2(\theta_2, X_2) f(\theta_2) d\theta_2$$

where we now write the firm's surplus in period 2 as $\Pi_2(\theta_2, X_2)$, with X_2 explicitly included as one of the arguments to reflect the fact that it depends on its stock at the beginning of period 2, as well as on θ_2 . The government's royalty in the second period is

$$R_2(\theta_2, X_2) = p_2 q_2^a(\theta_2, X_2) - C(q_2^a(\theta_2, X_2), \theta_2) - \Pi_2(\theta_2, X_2)$$

and its expected value is therefore

$$(27) \quad \Gamma^a(X_2) \equiv \int_{\theta^L}^{\theta^H} \left[p_2 q_2^a(\theta_2, X_2) - \theta_2 q_2^a(\theta_2, X_2) - \frac{b}{2} q_2^a(\theta_2, X_2)^2 \right] f(\theta_2) d\theta_2 - \Psi(X_2).$$

4.2. The First-Period Solution. Consider now the problem of period 1. The firm that reports $\hat{\theta}_1$ in period 1 will be asked by the government to produce $q_1(\hat{\theta}_1)$ and to pay $R_1(\hat{\theta}_1)$ in royalty. Its expected total discounted surplus if it reports $\hat{\theta}_1$

¹⁵ Again, $\underline{\theta}_2^a$ is a function of the endogenous value of X_2 . To simplify notation, we write it explicitly as such only when unavoidable.

when its true cost parameter in period 1 is θ_1 is therefore

$$p_1 q_1(\tilde{\theta}_1) - C(q_1(\tilde{\theta}_1), \theta_1) - R_1(\tilde{\theta}_1) + \delta \Psi(X_1 - q_1(\tilde{\theta}_1)).$$

Let $\phi(\tilde{\theta}_1; \theta_1)$ now represent this surplus and let $\hat{\Pi}(\theta_1) \equiv \phi(\theta_1; \theta_1)$. Again the values of $q_1(\tilde{\theta}_1)$ and of $R_1(\tilde{\theta}_1)$, and hence of $\hat{\Pi}(\theta_1)$, will also depend on X_1 .

As is shown in the Appendix, the government's problem in period 1 can be reduced to that of choosing $q_1(\theta_1)$ and $\hat{\Pi}(\theta^H)$ to maximize

$$(28) \quad V = \int_{\theta^L}^{\theta^H} \left\{ p_1 q_1(\theta_1) - \theta_1 q_1(\theta_1) - \frac{b}{2} q_1(\theta_1)^2 - (1 - \alpha) q_1(\theta_1) h(\theta_1) \right. \\ \left. + \delta [\Psi(X_1 - q_1(\theta_1)) + \Gamma^a(X_1 - q_1(\theta_1))] \right\} f(\theta_1) d\theta_1 - (1 - \alpha) \hat{\Pi}(\theta^H)$$

subject to $0 \leq q_1(\theta_1) \leq X_1$ and to $\hat{\Pi}(\theta^H) \geq 0$.

The maximization of V clearly requires $\hat{\Pi}(\theta^H) = 0$. Thus the optimal royalty schedule must leave no total discounted rent to the highest cost mine. The first-order condition with respect to q_1 is

$$(29) \quad p_1 - \theta_1 - b q_1 - (1 - \alpha) h(\theta_1) = \delta \left[\frac{d\Psi(X_1 - q_1)}{dX_2} + \frac{\Gamma^a(X_1 - q_1)}{dX_2} \right] \\ = \delta \int_{\theta^L}^{\theta_2^a} [p_2 - \theta_2 - b(X_1 - q_1)] f(\theta_2) d\theta_2$$

if we assume an interior solution for q_1 .

Define z_1^a by

$$(30) \quad p_1 - z_1^a - b X_1 - (1 - \alpha) g(z_1^a) = \delta \int_{\theta^L}^{\theta_2^a(q_1=X_1)} [p_2 - \theta_2] f(\theta_2) d\theta_2$$

where $\theta_2^a(q_1 = X_1)$ denotes the value of θ_2^a when $q_1 = X_1$, and y_1^a by

$$(31) \quad p_1 - y_1^a - (1 - \alpha) g(y_1^a) = \delta \int_{\theta^L}^{\theta_2^a(q_1=0)} [p_2 - \theta_2 - b X_1] f(\theta_2) d\theta_2$$

where $\theta_2^a(q_1 = 0)$ denotes the value of θ_2^a when $q_1 = 0$. If we set $\underline{\theta}_1^a = \min[z_1^a, \theta^H]$ and $\bar{\theta}_1^a = \max[y_1^a, \theta^L]$, and let $\tilde{q}_1^a(\theta_1)$ denote the solution to (29), then the optimal extraction rate for period 1, given the constraints $0 \leq q_1 \leq X_1$, may be written

$$(32) \quad q_1^a(\theta_1, X_1) = \begin{cases} X_1 & \text{if } \theta^L \leq \theta_1 \leq \underline{\theta}_1^a \\ \tilde{q}_1^a(\theta_1) & \text{if } \underline{\theta}_1^a < \theta_1 < \bar{\theta}_1^a \\ 0 & \text{if } \bar{\theta}_1^a \leq \theta_1 \leq \theta^H. \end{cases}$$

Note that in the case where $\underline{\theta}_2^a = \theta^H$ for all $X_2 \in [0, X_1]$ (i.e., $z_2^a \geq \theta^H$), so that the resource is exhausted in period 2 for all $\theta_2 \in [\theta_L, \theta^H]$, then

$$(33) \quad \tilde{q}_1^a(\theta_1) = \frac{p_1 - \theta_1 - (1 - \alpha)h(\theta_1) - \delta[p_2 - E\theta_2 - bX_1]}{b(1 + \delta)}.$$

Assumption (25) again guarantees that $q_1^a(\theta_1, X_1)$ is a nonincreasing function of θ_1 , thus satisfying condition (A.14), which is necessary for incentive compatibility.

4.3. The Modified Hotelling Rule. It was shown in Section 3 that the standard Hotelling rule must hold under symmetric information, in the form of condition (17). One implication of the Hotelling rule is that a positive discount rate ($\delta < 1$) will, ceteris paribus, result in a decreasing rate of extraction. This is immediately apparent from (17): if $p_2 = p_1$ and $E\theta_2 = \theta_1$, then $\delta < 1$ implies $q_1^s > q_2^s$.

To analyse the impact of asymmetric information on the optimal output path, it is useful to distinguish two cases. These cases arise under different values of the parameters. Let us call them case 1 and case 2.

Case 1. This case occurs whenever $p_2 \geq \theta^H + bX_1 + (1 - \alpha)h(\theta^H)$. Thus even under the most adverse cost conditions in period 2 ($\theta_2 = \theta^H$), adjusted marginal profit under asymmetric information would be nonnegative if the initial stock were to be totally extracted in that period.

Case 2. This case occurs whenever $p_2 < \theta^H + bX_1 + (1 - \alpha)h(\theta^H)$. Thus for firm type θ^H in period 2, adjusted marginal profit is negative at $q_2 = X_1$.

Recall that we have made at the outset the simplifying assumption that $p_2 \geq \theta^H + bX_1$, which guarantees that the resource stock is fully exhausted for all θ_2 under the reference scenario of symmetric information.

Consider now case 1. Clearly, in that case $z_2^a \geq \theta^H$ and hence $\underline{\theta}_2^a = \theta^H$ for all $X_2 \in [0, X_1]$. The optimal royalty scheme therefore requires that all firm types fully exhaust any remaining resource in period 2. In that case, the following proposition holds.

PROPOSITION 1. *If $\underline{\theta}_2^a = \theta^H$ for all $X_2 \in [0, X_1]$, then for any $\theta_1 \in [\theta_L, \theta^H]$, the optimal royalty scheme never requires that the optimal extraction path favor the present more under asymmetry of information than under symmetry of information.*

To prove this proposition, first note that

$$(34) \quad z_1^s - z_1^a = (1 - \alpha)g(z_1^a) - \delta \int_{\underline{\theta}_2^a(q_1=X_1)}^{\theta^H} [p_2 - \theta_2] f(\theta_2) d\theta_2$$

and

$$(35) \quad y_1^s - y_1^a = (1 - \alpha)g(y_1^a) - \delta \int_{\underline{\theta}_2^a(q_1=0)}^{\theta^H} [p_2 - \theta_2 - bX_1]f(\theta_2) d\theta_2$$

where $\underline{\theta}_2^s(q_1=0)$ and $\underline{\theta}_2^a(q_1=0)$ denote the values of $\underline{\theta}_2^s$ and $\underline{\theta}_2^a$ when $q_1=0$, while $\underline{\theta}_2^s(q_1=X_1)$ and $\underline{\theta}_2^a(q_1=X_1)$ denote the values of $\underline{\theta}_2^s$ and $\underline{\theta}_2^a$ when $q_1=X_1$. But with $\underline{\theta}_2^a = \theta^H$ for all $X_2 \in [0, X_1]$, as in case 1, (34) and (35) reduce to

$$(36) \quad z_1^s - z_1^a = (1 - \alpha)g(z_1^a) \geq 0$$

and

$$(37) \quad y_1^s - y_1^a = (1 - \alpha)g(y_1^a) \geq 0.$$

Therefore $\underline{\theta}_1^s - \underline{\theta}_1^a \geq 0$ and $\bar{\theta}_1^s - \bar{\theta}_1^a \geq 0$.

The inequality is strict in (36) and in (37) when respectively $\theta^L < z_1^a < \theta^H$ (some firms, but not all, exhaust the resource in period one) and $\theta^L < y_1^a < \theta^H$ (some firms, but not all, extract zero in period one) hold, thus whenever corner solutions occur in period 1 for some θ_1 . We then have $\underline{\theta}_1^s - \underline{\theta}_1^a > 0$ and $\bar{\theta}_1^s - \bar{\theta}_1^a > 0$.

It follows from these comparisons of the first-period optimal thresholds that both the upper extensive margin, beyond which firms are asked not to extract in the first period, and the lower extensive margin, below which firms are asked to exhaust the stock of resource in the first period, occur at levels of marginal cost not greater under asymmetry of information than under symmetry of information. Both of these margins in fact occur at strictly lower costs under asymmetry of information whenever corner solutions occur in period 1 for some θ_1 .

Therefore, whenever the optimal royalty scheme under asymmetric information requires some firm types to extract nothing in the first period, it requires more firm types to do so than under symmetric information. Similarly, whenever the optimal royalty scheme under asymmetric information requires some firm types to exhaust the resource stock in the first period, it requires fewer firm types to do so than under symmetric information.

Furthermore, from the interior solutions (16) and (33) we verify that when $\underline{\theta}_2^a = \theta^H$ for all $X_2 \in [0, X_1]$, as in case 1,

$$(38) \quad \begin{aligned} \bar{q}_1^s(\theta_1) &= \bar{q}_1^a(\theta_1) + \frac{(1 - \alpha)h(\theta_1)}{(1 + \delta)b} \\ &> (=) \bar{q}_1^a(\theta_1) \quad \text{for } \theta_1 > (=) \theta^L. \end{aligned}$$

We may therefore write the difference between the solution under symmetry and

that under asymmetry of information as

$$(39) \quad q_1^s(\theta_1, X_1) - q_1^a(\theta_1, X_1) = \begin{cases} 0 & \text{if } \theta^L \leq \theta_1 \leq \underline{\theta}_1^a \\ X_1 - \bar{q}_1^a(\theta_1) > 0 & \text{if } \underline{\theta}_1^a < \theta_1 \leq \underline{\theta}_1^s \\ \bar{q}_1^s(\theta_1) - \bar{q}_1^a(\theta_1) > 0 & \text{if } \underline{\theta}_1^s < \theta_1 \leq \bar{\theta}_1^a \\ \bar{q}_1^s(\theta_1) > 0 & \text{if } \bar{\theta}_1^a \leq \theta_1 < \bar{\theta}_1^s \\ 0 & \text{if } \bar{\theta}_1^s \leq \theta_1 \leq \theta^H. \end{cases}$$

Proposition (1) follows directly from (39).

Figure 1 illustrates the relationship between the two solutions as a function of θ_1 when corner solutions occur in period 1. The solution under symmetry of informa-

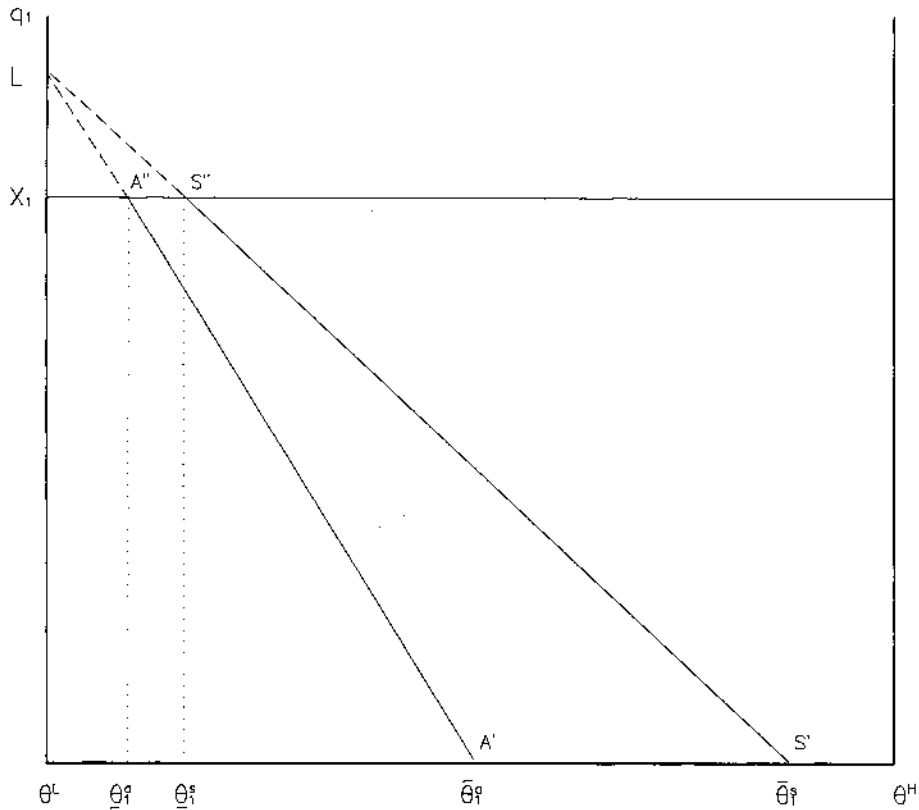


FIGURE 1

$\theta_2^a = \theta^H$ FOR ALL $X_2 \in [0, X_1]$ AND CORNER SOLUTIONS IN PERIOD 1

tion is given by the locus $X_1 S'' S'$ and zero thereafter, whereas the solution under asymmetry of information is represented by the locus $X_1 A'' A'$ and zero thereafter.

It is interesting to look more closely at the situation where, contrary to what is depicted in Figure 1, interior solutions occur for both $q_1^s(\theta_1, X_1)$ and $q_1^a(\theta_1, X_1)$ for all $\theta_1 \in [\theta^L, \theta^H]$. This arises when $y_1^s \leq \theta^L$, so that $\theta_1^s = \theta^L$, and $z_1^s \geq \theta^H$, so that $\theta_1^s = \theta^H$.

The interior solution under asymmetric information, $\bar{q}_1^a(\theta_1)$, is the solution to (29). But with $\theta_2^a = \theta^H$ for all $X_2 \in [0, X_1]$, condition (29) becomes simply

$$(40) \quad p_1 - MC(q_1, \theta_1) - (1 - \alpha)h(\theta_1) = \delta[p_2 - EMC(X_1 - q_1, \theta_2)].$$

This is the Hotelling rule modified to account for the informational constraints resulting from the asymmetry of information. It says that the marginal profit properly corrected for the cost of the informational constraint, must grow at the rate of interest.

Notice that since $h(\theta^L) = 0$, the standard Hotelling rule is unmodified for the lowest-cost firm. But all higher cost firms are asked to produce less in period 1 than under symmetry of information, and hence more in period 2, since $q_2 = X_1 - q_1$ by the assumption of case 1. For all those firms, the decline in the extraction path is therefore strictly less pronounced than under symmetric information. In fact, if the weight given to the rent left in the hands of the mining firms is sufficiently small, i.e., if, for a given θ_1 , $(1 - \alpha)$ is sufficiently large, future extraction may be favored over present extraction, thus resulting in an increasing extraction path. This distortion to the standard Hotelling rule is necessary in order to provide the incentive for firms not to exaggerate their costs. The distortion is in fact similar to that which is required in the usual static problems of this type, in that it perturbs all but the most efficient firm.

Figure 2 depicts the situation where only strictly interior solutions occur. The "LS" locus designates the equilibrium solution as a function of θ_1 in the case of symmetric information (i.e., the solution to (17)), whereas the "LA" locus is its asymmetric information counterpart. The decline in the extraction path under symmetric information is reflected in the fact that $q_1^s > X_1/2$ for $\theta_1 = E\theta_2$.

All the above results, including the result that the Hotelling rule is unmodified for the least-cost firm, depend crucially on the fact that the initial resource stock is exhausted by the end of the final period for all θ_2 in both the asymmetric and the symmetric information situations. Thus the results depend crucially on the assumption of case 1.

Consider now case 2. In case 2, the optimal extraction program from the government's point of view requires that for some θ_2 and some X_2 the resource stock not be exhausted in period 2 under asymmetric information. Unlike in case 1, a change in the quantity extracted in the first period will not then necessitate an equivalent change in the opposite direction in the second period, since the resource constraint is not binding. It is important to remember this when comparing the first-period extraction under asymmetric information to that under symmetric information.

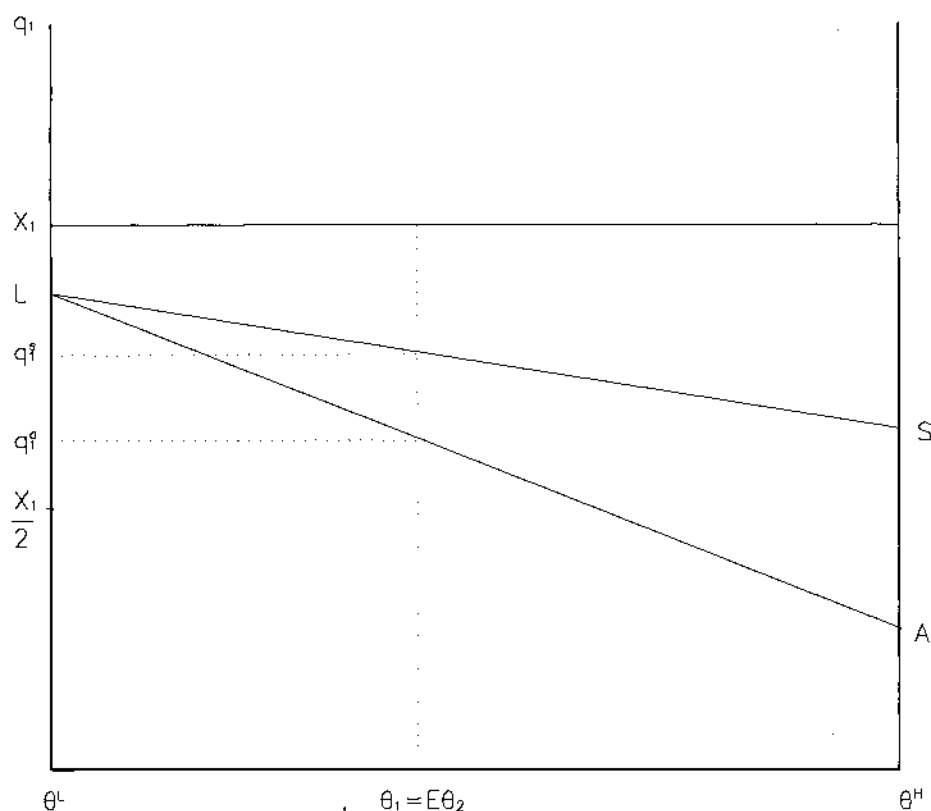


FIGURE 2

$\theta_2^a = \theta^H$ FOR ALL $X_2 \in [0, X_1]$ AND NO CORNER SOLUTIONS IN PERIOD 1

Since in that case we have $\theta_2^a < \theta^H$ for some $X_2 \in [0, X_1]$, then, from (16) and (29),

$$(41) \quad \bar{q}_1^i(\theta_1) = \bar{q}_1^a(\theta_1) + \frac{Z(\theta_1)}{(1+\delta)b}$$

where

$$(42) \quad Z(\theta_1) = (1-\alpha)h(\theta_1) - \delta \int_{\theta_1^a}^{\theta^H} [p_2 - \theta_2 - b(X_1 - \bar{q}_1^a(\theta_1))] f(\theta_2) d\theta_2.$$

The integral on the right-hand side of (42) is nonnegative since $p_2 - bX_1 \geq \theta^H$. It follows that $Z(\theta_1) \leq (1-\alpha)h(\theta_1)$ and hence, comparing (41) and (38) we have the following.

PROPOSITION 2. *When the optimal royalty schedule under asymmetry of information requires of some firms that they not exhaust their remaining resource stock in the second period (i.e., when $\underline{\theta}_2^a < \theta^H$ for some $X_2 \in [0, X_1]$), it will tend to require less reduction in the first-period extraction than when all firms are required to exhaust their second-period stock (i.e., when $\underline{\theta}_2^a = \theta^H$ for all $X_2 \in [0, X_1]$).*

The reason for this result is that when the optimal royalty under asymmetric information requires some of the firms not to exhaust their resource stocks, the opportunity cost of extracting the resource today rather than leaving it in the ground for future extraction is, *ceteris paribus*, lower than it would otherwise be and lower than under symmetric information. A greater part of the divergence between first-period price and first-period marginal extraction cost is now attributable strictly to the informational constraints.

In fact, the optimal royalty in that case may require any given firm type to extract more in the first period than it would under symmetric information. Consider in particular the lowest-cost firm in period 1, namely the firm of type $\theta_1 = \theta^L$. Since $h(\theta^L) = 0$, it follows from (41) that for any $\tilde{q}_1^a(\theta^L) > 0$, $\tilde{q}_1^a(\theta^L) > \tilde{q}_1^s(\theta^L)$. Thus, assuming that not all firms were asked to exhaust their stock in the first period under both symmetric and asymmetric information, Proposition 3 follows.

PROPOSITION 3. *When $\underline{\theta}_2^a < \theta^H$ for some $X_2 \in [0, X_1]$, so that some firms are required not to exhaust their remaining resource stock in the second period, the optimal royalty under asymmetry of information always requires a distortion in the extraction path of the lowest-cost firms: they must extract more than under symmetric information.*

This contrasts with the standard results concerning incentive schemes in a static relationship or a multiperiod relationship without constraints, which invariably state that it is never optimal to induce distortion on the lowest-cost agent. Our result indicates that when an agent's decision rule in period 2 depends on the stock left over from period 1, then it is in general optimal to induce distortion in period 1, even for the lowest-cost agent.

The intuition is similar to Proposition 2. In case 2 the information constraints induce some inefficiency in the second period, in the form of lower production than under symmetric information for some of the higher cost firms. The foregone future net benefits of having the lowest-cost firm extract more in the first period are therefore lower under asymmetry of information. Thus, although no distortion is required for the lowest cost firm because of first period asymmetry of information (i.e., $h(\theta^L) = 0$), it is optimal to have it produce at a higher rate than under symmetric information.

This will not occur in case 1, since in that case no distortion is introduced in the second period, all the firms being required to exhaust the remaining resource stock under both asymmetric and symmetric information. However the results of Proposition 2 become the rule rather than the exception whenever the optimal terminal period exceeds period 2. For then, even though the information constraints may not

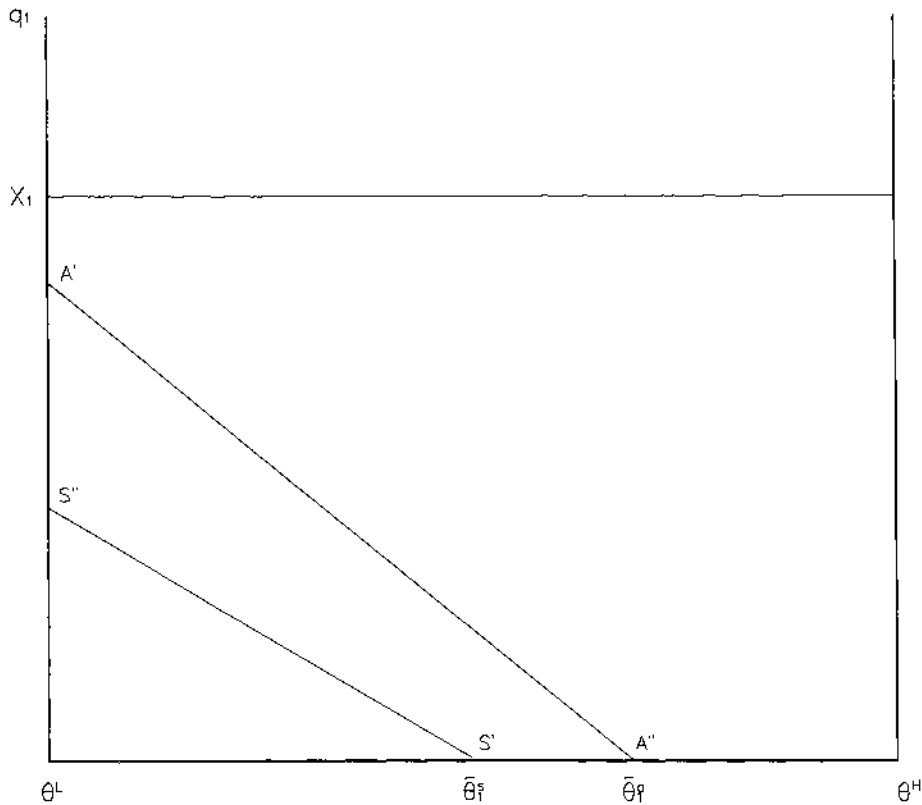


FIGURE 3

$$\underline{\theta}_1^A < \theta^H \text{ FOR SOME } X_2 \in [0, X_1]$$

induce any distortions in the final period, as in case 1, by Proposition 1 they always will for some firm types in the next to last period and, a fortiori, in preceding periods. Since distortions will be required in all periods intermediate between the current and the last, the opportunity cost of extracting the resource in the current period rather than leaving it for future extraction will be different under asymmetric information than under symmetric information and the output path of the lowest-cost firm will always be affected by the optimal incentive scheme. This will become apparent when we discuss the effect of asymmetry of information on the optimal terminal period in the next section.

Although it cannot be ruled out that less efficient firms may still be asked to extract less in period 1 than under symmetry, the situation depicted in Figure 3 is easily conceivable in the two periods scenario. It illustrates a case where some firms are required not to exhaust the remaining stock in period 2 and some firms are required to produce more in the first period than under symmetric information. It should be remembered that contrary to Figures 1 and 2, a firm's second-period production under asymmetric information cannot be derived as the residual $X_1 - q_1^q$

in Figure 3, although its second-period production under symmetry is still given by $X_1 - q_1^s$.

5. THE OPTIMAL TERMINAL PERIOD

We have thus far assumed that the resource is worthless for $t = 3, 4, \dots$. It is therefore never optimal to exploit the resource beyond the second period, whether information is symmetric or asymmetric and whether all firms are required to exhaust their resource stock or not. Although $p_t = 0$ for all $t > 2$ is sufficient for this to hold, it is not necessary. And once we relax this assumption, it becomes possible to analyze the effect of the asymmetry in information on the duration of the relationship, in addition to the effect on the tilt of the extraction path discussed in the previous sections. It turns out that the effect on the duration is in general ambiguous. Our analysis will also demonstrate the fact that the optimal royalty scheme will, in the general case, always distort the choice of even the lowest cost firm, in contrast with optimal static incentive mechanisms. Therefore the case encountered in the previous section where the choice of the lowest cost firm was left undistorted—when all firm types leave nothing unexploited by the end of the second period and all first period solutions are interior—was in fact an artifice of the two period case.

A generalization of the method of solution employed in Section 4 to an arbitrary number of periods is presented in the Appendix. We will in this section, as in the Appendix, write the functions $\Gamma(\cdot)$ and $\Psi(\cdot)$ as $\Gamma_t(\cdot)$ and $\Psi_t(\cdot)$, with $t = 2, 3, \dots$, in order to allow for the possibility that the optimal contingent plan beyond period 1 may contain more than one period. We also introduce the notations T^s and T^a for the optimal terminal period under, respectively, symmetric and asymmetric information. The conditions which define the optimal terminal period are given in the Appendix.

To analyze the effect of asymmetry of information on duration, it will suffice to allow for the possibility that a third period be added endogenously. Thus suppose now $p_3 \geq 0$ and $p_t = 0$ for all $t > 3$, so that it may become optimal, for some p_3 , to prolong extraction beyond period 2. In order to make this prospect potentially attractive for all firm types, assume further that

$$(43) \quad p_3 - \theta^H - bX_1 \geq 0.$$

This last assumption says that under symmetric information, even the highest cost firm would find it optimal to exploit the total initial stock during the third period, if faced with that prospect. This guarantees that no firm type will wish to leave part of the resource stock unexploited under symmetric information.

Assume further that the parameters X_1 , p_1 , p_2 and p_3 are such that all firm types find it optimal to choose period 2 as the optimal terminal period under symmetric information, whatever the sequence $\{\theta_1, \theta_2\}$. This assumption allows us to investigate the possibility of a reduction ($T^a = 1$) or an increase ($T^a = 3$) of the terminal period. We will show, via examples, that both cases may arise.

If as assumed $T^s = 2$, then it follows from (43) that the contingent plan for period 2 simply consists in setting $q_2^s = X_2^s = X_1 - q_1^s$, for otherwise period 3 would be the optimal terminal period for some θ_2 . It also follows that no firm type must find it optimal to set $q_1^s = X_1$, for otherwise period 1 would be the terminal period for those firm types.

A necessary condition for this last statement to hold is that for all firm types, the marginal surplus from setting $q_1^s = X_1$ be lower than the discounted expected marginal surplus from optimally exploiting a marginal unit of reserves in period 2. Hence there must exist some $\epsilon > 0$ such that

$$(44) \quad p_1 - \theta^L - bX_1 + \epsilon = \delta[p_2 - E\theta_2] \left(= \delta \frac{d\Gamma_2^s(0)}{dX_2} \right).$$

Notice that the way the right-hand side of (44) is written depends crucially on the fact that period 2 is the optimal terminal period, by assumption. For otherwise it should reflect the discounted expected surplus from exploiting a marginal unit according to the optimal contingent plan beginning in period 2 and possibly ending in period 3.

Another necessary condition for period 2 to be the optimal terminal period under symmetric information, in addition to (44), is that for all firm types and for any X_2 that may have been kept after period 1, the marginal surplus from exhausting in period 2 not be lower than the expected marginal surplus from optimally exploiting a marginal unit of the resource in period 3. Hence we must have

$$(45) \quad p_2 - \theta^H - bX_2^s(\theta^H) \geq \delta[p_3 - E\theta_3] \left(= \frac{d\Gamma_3^s(0)}{dX_3} \right),$$

where $X_2^s(\theta^H) = X_1 - q_1^s(\theta^H)$. The right-hand side of (45) reflects the fact that because of assumption (43), $\theta_3^s = \theta^H$ for all $X_3^s = X_2^s - q_2^s$. Together, conditions (44) and (45) are necessary and sufficient for $T^s = 2$ for all sequences of firm types.

To show that asymmetry of information may strictly lengthen the duration of exploitation, assume that (45) holds with equality and that $p_1 = 0$, so that $X_2^s = X_2^a = X_1$ for all $\theta_1 \in [\theta^L, \theta^H]$. Hence (45) may be written

$$(46) \quad p_2 - \theta^H - bX_1 = \delta[p_3 - E\theta_3].$$

Now a sufficient condition for some firm type not to exhaust in period 2 under asymmetry of information is that there exists some $\theta_2 \in [\theta^L, \theta^H]$ such that

$$(47) \quad p_2 - \theta_2 - bX_1 - (1 - \alpha)h(\theta_2) < \delta[p_3 - E\theta_3].$$

Subtracting (46) from (47), we find that (47) is satisfied for any $\theta_2 > \hat{\theta}$, where $\hat{\theta}$ is defined by $\theta^H - \hat{\theta} - (1 - \alpha)h(\hat{\theta}) = 0$. Hence these firms will have some positive

stock of the resource remaining in period 3 and it is sufficient that $p_3 - \theta^H > 0$ in order for them to find it profitable to operate in period 3, which is the case when (43) holds. Since this last condition is compatible with (47) and (46), we have an example where $T^a > T^s$.

Consider now an example where asymmetry of information results in $T^a = 1$, hence strictly shortening the duration of exploitation. If period 1 is the optimal terminal period under asymmetric information for some firm type, then that type will either exhaust in period 1 or, if it does not, it will not find it profitable, no matter what the realized sequence $\{\theta_2, \theta_3\}$, to produce in periods 2 and 3. Otherwise, by definition, period 1 would not have been the optimal terminal period. In both instances, it is necessary and sufficient that the marginal surplus from period 1 production be at least as high as the discounted expected marginal surplus, $d\Gamma_2^a/dX_2 + d\Psi_2/dX_2$, from optimally exploiting a marginal unit of reserves in period 2, namely (see (A.25) in the Appendix)

$$(48) \quad p_1 - \theta_1 - bq_1^a(\theta_1, X_1) - (1 - \alpha)h(\theta_1) \geq \delta \left[\frac{d\Gamma_2^a(0)}{dX_2} + \frac{d\Psi_2(0)}{dX_2} \right]$$

with $d\Gamma_2^a(0)/dX_2 + d\Psi_2(0)/dX_2 = 0$ if $q_1^a(\theta_1, X_1) < X_1$.

Now assume for example that $\theta_1 = \theta^L$ and therefore $h(\theta^L) = 0$. Then, considering (44), condition (48) will be satisfied if

$$\frac{d\Gamma_2^a(0)}{dX_2} + \frac{d\Psi_2(0)}{dX_2} \leq \frac{d\Gamma_2^s(0)}{dX_2} - \frac{\epsilon}{\delta},$$

or, taking ϵ arbitrarily small,

$$\frac{d\Gamma_2^a(0)}{dX_2} + \frac{d\Psi_2(0)}{dX_2} < \frac{d\Gamma_2^s(0)}{dX_2}.$$

From (A.22) (see the Appendix) and the fact that $d\Gamma_2^s(0)/dX_2 = p_2 - E\theta_2$ since $T^s = 2$ by assumption, this is equivalent to

$$\begin{aligned} & \int_{\theta^L}^{\bar{\theta}_2^a(q_1=X_1)} [p_2 - \theta_2] f(\theta_2) d\theta_2 + \delta \left[\frac{d\Gamma_3^a(0)}{dX_3} + \frac{d\Psi_3(0)}{dX_3} \right] \\ & \times [1 - F(\bar{\theta}_2^a(q_1=X_1))] > p_2 - E\theta_2 \end{aligned}$$

which we may rewrite

$$(49) \quad \begin{aligned} & \int_{\bar{\theta}_2^a(q_1=X_1)}^{\theta^H} \delta \left[\frac{d\Gamma_3^a(0)}{dX_3} + \frac{d\Psi_3(0)}{dX_3} \right] f(\theta_2) d\theta_2 \\ & < \int_{\bar{\theta}_2^a(q_1=X_1)}^{\theta^H} [p_2 - \theta_2] f(\theta_2) d\theta_2. \end{aligned}$$

Using again (A.22), with $d\Gamma_{t+1}^a(0)/dX_{t+1} + d\Psi_{t+1}(0)/dX_{t+1} = 0$ for all $t \geq 3$ since $p_t = 0$ for all $t > 3$ by assumption, we verify that

$$\frac{d\Gamma_3^a(0)}{dX_3} + \frac{d\Psi_3(0)}{dX_3} \leq p_3 - E\theta_3.$$

Since by condition (45) we have

$$\delta[p_3 - E\theta_3] < p_2 - \theta_2$$

it follows that (5) is satisfied and therefore $T^a = 1 < T^s$ for $\theta_1 = \theta^L$.

In addition to providing an example where the duration of extraction is strictly shortened by the presence of asymmetry of information, the above analysis also shows that distortion of the lowest-cost firm under the optimal incentive scheme with asymmetric information is a robust result in an inherently dynamic framework. The intuition behind this result can be seen by considering condition (A.24) (see Appendix), the first-order condition for an interior solution under asymmetry of information, which we rewrite

$$(50) \quad p_t = MC(q_t, \theta_t) + (1 - \alpha)h(\theta_t) + \delta \left[\frac{d\Gamma_{t+1}^a(X_{t+1}^a)}{dX_{t+1}^a} + \frac{d\Psi_{t+1}(X_{t+1}^a)}{dX_{t+1}^a} \right].$$

This is the familiar "price = marginal cost" rule, where the cost side of the equation takes into account the full opportunity cost. The latter is composed of three components corresponding to the three terms on the right-hand side of (50). The first component, $MC(q_t, \theta_t) = \theta_t + bq_t$, is the usual marginal cost of production—in this case the marginal cost of extracting the resource. The second component may be called the current marginal cost of incentive compatibility. As is well known, it vanishes whenever $\theta_t = \theta^L$. This explains why the lowest-cost firm is not affected by the optimal incentive mechanism when the third component is not present, as in static models. This last component is the discounted expected marginal surplus from the optimal contingent program beginning in period $t + 1$. It captures the intertemporal opportunity cost of shifting production at the margin from the future to the current period. It is due to the inherently dynamic nature of the problem and is present under both symmetric and asymmetric information. Since the optimal incentive scheme reduces expected total surplus, the value of this component of marginal cost under asymmetry of information will generally differ from the value of its counterpart under symmetric information.

The optimal incentive mechanism reduces (increases) current production when the sum of the current marginal cost of incentive compatibility and the *change* in the intertemporal opportunity cost is positive (negative). For a sequence of relatively high θ_t 's, the first effect is more likely to dominate, as in our first example, where $T^a > T^s$. For the most favourable sequence of realizations, $\{\theta_t = \theta^L | t =$

$1, 2, \dots\}$, $h(\theta^L) = 0$ at all periods and the second effect dominates. This explains our second example, where $T^a < T^s$.

One can of course construct examples of "degenerate corner-solutions" where the lowest-cost firm remains unperturbed under asymmetry of information. One such example occurs in Section 4, in the case where the price sequence is such that all firm types are required not to leave any stock unexploited past the second period under both asymmetric and symmetric information. The production path of the lowest-cost firm is then left unchanged. Since the price sequence is in that case assumed such that *all* firm types are required to exhaust their remaining stock in period 2, there is no incentive problem left in the second period. As a result, the intertemporal opportunity cost component is identical under both asymmetric and symmetric information. Since at most one period precedes the terminal period, it follows from (50) that $q_1^s(\theta^L, X_1) = q_1^s(\theta^L, X_1)$ and, exceptionally, the two output paths coincide for $\theta_1 = \theta^L$.

6. THE OPTIMAL ROYALTY

We now discuss briefly the determination of the royalty rule, expressed as a function of the rate of extraction, which will realize the optimal program just described. We will denote this function $R_t^*(q_t)$. It is well known that there exists such a payment scheme which is equivalent to the optimal revelation mechanism. It will suffice here to illustrate with the case where period 2 is the optimal terminal period. This will allow us to show in particular how the government will capture some of the expected second period rent via a lump sum payment in the first period. Without loss of generality, we assume in this section that $\theta^L = 0$ or, alternatively, that prices are net of θ^L .

Consider first the second-period royalty. From the incentive compatibility constraint (A.4), we derive that

$$(51) \quad \frac{dR_2}{dq_2} = p_2 - \theta_2 - bq_2$$

whenever $dq_2/d\theta_2 < 0$. We know this is true for $q_2(\theta_2)$ satisfying equation (20).

Now denote by $\Theta_2(q_2)$ the value of θ_2 which exactly solves equation (20). Substituting for θ_2 into (51) and using the fact that $\int_0^{q_2} \min[\Theta_2(\tau), \theta^H] d\tau = \int_{q_2(\theta^H)}^{q_2} \Theta_2(\tau) d\tau + \theta^H q_2(\theta^H)$, we find that

$$(52) \quad R_2^*(q_2) = p_2 q_2 - \frac{b}{2} q_2^2 - \int_{q_2(\theta^H)}^{q_2} \Theta_2(\tau) d\tau - \theta^H q_2(\theta^H).$$

The boundary condition $\Pi_2(\theta^H) = 0$ has been used to eliminate the constant of integration in (52).

It is easy to verify that a firm of type $\theta_2 \in [\theta^L, \theta^H]$, faced with this royalty scheme and choosing $q_2 \in [0, X_2]$ to maximize its profit net of the royalty, will choose to

extract a quantity

$$(53) \quad q_2(\theta_2, X_2) = \begin{cases} X_2 & \text{if } \theta^L \leq \theta_2 \leq \Theta_2(X_2) \\ \tilde{q}_2^a(\theta_2) & \text{if } \Theta_2(X_2) < \theta_2 < \Theta_2(0) \\ 0 & \text{if } \Theta_2(0) \leq \theta_2 \leq \theta^H \end{cases}$$

which is exactly $q_2^a(\theta_2, X_2)$, the desired result.

To illustrate this royalty function with an example, assume θ_2 distributed uniformly over $[0, 1]$. Then $h(\theta_2) = \theta_2$, $\Theta_2(q_2) = [p_2 - bq_2]/(2 - \alpha)$, and

$$(54) \quad R_2^*(q_2) = \frac{(1 - \alpha) \left[p_2 q_2 - \frac{b}{2} q_2^2 \right]}{2 - \alpha} + A_2(q_2(\theta^H))$$

where

$$(55) \quad A_2(q_2(\theta^H)) = \frac{p_2 - (2 - \alpha)\theta^H}{2 - \alpha} q_2(\theta^H) - \frac{b}{2(2 - \alpha)} q_2(\theta^H)^2.$$

By a similar method, we can determine the optimal royalty schedule $R_1^*(q_1)$ for period 1. Consider for a moment $q_1(\theta_1)$ which solves equation (29). Since it satisfies $dq_1/d\theta_1 < 0$, we find from (A.13) that

$$(56) \quad \frac{dR_1}{dq_1} = p_1 - \theta_1 - bq_1 - \delta \frac{d\Psi(X_1 - q_1)}{dX_2}$$

where $d\Psi(X_1 - q_1)/dX_2$ is the second period expected marginal profit, itself derived from (26). Now let $\Theta_1(q_1)$ denote the value of θ_1 that solves equation (29). Substituting for θ_1 in (56) and using the fact that $\int_{q_1(\theta^H)}^{q_1} \min[\Theta_1(\tau), \theta^H] d\tau = \int_{q_1(\theta^H)}^{q_1} \Theta_1(\tau) d\tau + \theta^H q_1(\theta^H)$, we find that the first-period royalty can be written

$$(57) \quad R_1^*(q_1) = p_1 q_1 - \frac{b}{2} q_1^2 - \int_{q_1(\theta^H)}^{q_1} \Theta_1(\tau) d\tau - \theta^H q_1(\theta^H) + \delta [\Psi(X_1 - q_1) - \Psi(X_1)] + K.$$

In order to determine the constant of integration, we now make use of the boundary condition $\hat{\Pi}(\theta^H) = 0$. Since $\hat{\Pi} = \Pi_1 + \delta E\Pi_2$, we imply from this condition that

$$\Pi_1(\theta^H) = -\delta \Psi(X_1 - q_1(\theta^H)).$$

Hence, from the definition of $R_1(\theta_1)$,

$$(58) \quad R_1(\theta^H) = p_1 q_1(\theta^H) - \theta^H q_1(\theta^H) - \frac{b}{2} q_1(\theta^H)^2 + \delta \Psi(X_1 - q_1(\theta^H)).$$

Equating (57) evaluated at θ^H with (58), we can solve for K to yield $K = \delta \Psi(X_1)$.

As for period 2, one can easily verify that when faced with this royalty rule, the firm of type θ_1 will choose to extract in period 1 exactly the quantity of the resource which satisfies the government's optimal program, namely $q_1^2(\theta_1, X_1)$.

To illustrate this royalty schedule, consider the special case where $q_2 = X_1 - q_1$ for all $\theta_2 \in [\theta^L, \theta^H]$ (i.e., all firms are required to exhaust in period 2). In that case $\Psi(X_1 - q_1) = (\theta^H - E\theta_2)[X_1 - q_1]$. If we further assume that θ_1 is distributed uniformly over $[0, 1]$, then

$$\Theta_1(q_1) = \frac{p_1 - (1 + \delta)bq_1 - \delta[p_2 - E\theta_2 - bX_1]}{2 - \alpha}$$

and the first-period royalty is given by

$$(59) \quad R_1^*(q_1) = \delta \frac{X_1}{2} + \frac{(1 - \alpha)p_1 - \delta(2 - \alpha)/2 + \delta(p_2 - E\theta_2 - bX_1)}{2 - \alpha} q_1 \\ - \frac{(1 - \alpha - \delta)b}{2(2 - \alpha)} q_1^2 + A_1(q_1(\theta^H))$$

where

$$(60) \quad A_1(q_1(\theta^H)) = \frac{p_1 - \delta[p_2 - E\theta_2 - bX_1] - (2 - \alpha)\theta^H}{2 - \alpha} q_1(\theta^H) \\ - \frac{(1 + \delta)b}{2(2 - \alpha)} q_1(\theta^H)^2.$$

The terms $A_1(q_1(\theta^H))$ and $A_2(q_2(\theta^H))$ in (59) and (54), or their implicit analogues in the general royalty functions (57) and (52), are designed to capture the remaining rent, if any, of the least efficient firms—firms of type θ^H —in periods 1 and 2 respectively. They are zero if respectively $q_1(\theta^H) = 0$ and $q_2(\theta^H) = 0$. Notice however that in that case $R_2^*(0)$ is zero, but $R_1^*(0)$ remains positive. Thus, contrary to the second-period royalty, the first-period royalty is always composed of a fixed

amount plus some function of q_1 .¹⁶ The reason is that although the contract between the government and the firm is a two-period contract, the firm can opt out in the second period and thus is not obliged to pay the government anything in that period if its θ_2 is unfavorable. Therefore its expected rent in period 2 is positive. By setting $R_1^*(0) > 0$, the government is able to capture some of that rent in the first period. The government only has to make sure in determining this fixed amount that the negative surplus which results in the first period for some firms is sufficiently compensated by the positive expected profit in the second period to satisfy the first period participation constraint.

7. CONCLUSION

This paper has shown how the presence of asymmetry of information concerning the structure of extraction costs constrains the government's effort to recuperate the rent from a stock of nonrenewable natural resource via a royalty payment imposed on the firms exploiting the resource. When the institutional arrangements are such that the government relies on royalty contracts, the observed extraction path will not generally satisfy the usual Hotelling arbitrage rule, which would hold under symmetric information. This is due to the need to provide the firms with an incentive not to exaggerate their costs of extraction when adverse selection is present. If all firm types are required to exhaust their resource stock by the end of the second period, the optimal royalty scheme will generate an extraction path which favors first-period production less than under symmetric information. This effect can be reversed when it is optimal to have some firm types leave part of their resource stock unexploited.

We have also shown that when we generalize to an arbitrary number of periods, the presence of asymmetric information has an ambiguous effect on the optimal terminal period. The optimal incentive scheme under asymmetry of information also has the effect of imposing a distortion on the output path of the lowest-cost firm. This important property of the dynamic framework is in contrast with the static model, in which the lowest-cost firm always remains efficient under asymmetric information.

These results have been obtained under the assumption that the variable of adverse selection is not correlated over time. This assumption may realistically capture some situations and is useful to focus attention on the dynamics imposed by the nonrenewability constraint in a context of adverse selection. Quite obviously, however, it would be important to relax it in future work in order to arrive at a more general characterization of the optimal royalty. It would also be interesting to consider the effect of asymmetry of information on other parameters, such as, for

¹⁶ As the example in (59) illustrates, the curvature of this function will depend on the rate of discount and the weight given to the rent left in the hands of the firms. In the particular example, it will be strictly concave (strictly convex) whenever $(1 - \alpha - \delta) > (<) 0$, and linear whenever $(1 - \alpha - \delta) = 0$.

example, the size of the resource stock. Finally, similar results should also apply to other dynamic situations.

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APPENDIX

A. Characterization of Incentive Compatible Mechanisms. In this appendix section we first characterize the class of incentive compatible mechanisms, that is mechanisms in response to which the firm will choose to reveal its true cost parameter.

A.1. Period 2. Consider the last period first. Let $\phi(\tilde{\theta}_2; \theta_2)$ be the surplus of the firm in period 2 if it reports $\tilde{\theta}_2$ when the true cost parameter takes the value θ_2 . The government asks the firm that reports $\tilde{\theta}_2$ to produce $q_2(\tilde{\theta}_2)$ and to pay the government the total royalty $R_2(\tilde{\theta}_2)$. Hence

$$(A.1) \quad \phi(\tilde{\theta}_2; \theta_2) = p_2 q_2(\tilde{\theta}_2) - \theta_2 q_2(\tilde{\theta}_2) - \frac{b}{2} q_2(\tilde{\theta}_2)^2 - R_2(\tilde{\theta}_2).$$

For the firm to respond truthfully, it is necessary that, for all $\theta_2 \in [\theta^L, \theta^H]$,

$$(A.2) \quad \phi_1(\tilde{\theta}_2; \theta_2) = 0 \quad \text{for } \tilde{\theta}_2 = \theta_2$$

and

$$(A.3) \quad \phi_{11}(\tilde{\theta}_2; \theta_2) \leq 0 \quad \text{for } \tilde{\theta}_2 = \theta_2.$$

Condition (A.2) implies that the incentive scheme must satisfy

$$(A.4) \quad \left[p_2 - \theta_2 - b q_2(\tilde{\theta}_2) \right] \frac{d q_2(\tilde{\theta}_2)}{d \tilde{\theta}_2} - \frac{d R_2(\tilde{\theta}_2)}{d \tilde{\theta}_2} = 0 \quad \text{for } \tilde{\theta}_2 = \theta_2.$$

In addition, since by condition (A.2) $\phi_1(\theta_2; \theta_2) = 0$ for all $\theta_2 \in [\theta^L, \theta^H]$, we have

$$\phi_{11}(\theta_2; \theta_2) + \phi_{12}(\theta_2; \theta_2) = 0 \quad \forall \theta_2 \in [\theta^L, \theta^H].$$

It follows that condition (A.3) is equivalent to

$$(A.5) \quad \phi_{12}(\theta_2; \theta_2) = -\frac{d q_2}{d \theta_2} \geq 0.$$

This condition says that, for incentive compatibility, high cost firms must not be asked to extract more of the resource than low cost firms.

Conditions (A.4) and (A.5) are local conditions. However, given the linearity of the cost function in θ_2 , they are sufficient for global incentive compatibility to hold. (See for example Baron 1989 for the method of proof, or Gaudet, Lasserre, and Long 1991 for its details in the particular context discussed here.)

Note next that if we let $\Pi_2(\theta_2) \equiv \phi(\theta_2; \theta_2)$, then, by the envelope theorem, we must have

$$(A.6) \quad \frac{d\Pi_2}{d\theta_2} = \phi_2(\theta_2; \theta_2) = -q_2(\theta_2).$$

Therefore $\Pi_2(\theta_2)$ is a nonincreasing function of θ_2 : an incentive compatible mechanism must not leave low-cost firms with a smaller surplus than high-cost firms. If we integrate condition (A.6), it may, as an alternative, be stated as a condition on the profit function itself:

$$(A.7) \quad \Pi_2(\theta_2) = \Pi_2(\theta^H) + \int_{\theta_2}^{\theta^H} q_2(\tau) d\tau.$$

Condition (A.6)—or (A.7)—is a local condition. It must hold in a neighborhood of $\bar{\theta}_2 = \theta_2$.

Finally, it is assumed that the firm can decide to opt out at each period in response to the announced incentive scheme. This means that the combined royalty and production schedules must satisfy a participation constraint at each period. Thus for period 2, we must have

$$(A.8) \quad \Pi_2(\theta_2) \geq 0 \quad \forall \theta_2 \in [\theta^L, \theta^H].$$

This set of constraints simply requires that the incentive scheme guarantee each firm a nonnegative surplus. Notice that because $\Pi_2(\theta_2)$ is a nonincreasing function of θ_2 by (A.6), the constraints (A.8) may be replaced by the single constraint

$$(A.9) \quad \Pi_2(\theta^H) \geq 0.$$

The government's problem can now be stated as choosing $\{(R_2(\theta_2), q_2(\theta_2)) | \theta_2 \in [\theta^L, \theta^H]\}$ to maximize

$$(A.10) \quad EW_2 = \int_{\theta^L}^{\theta^H} [R_2(\theta_2) + \alpha \Pi_2(\theta_2)] f(\theta_2) d\theta_2$$

subject to (A.4), (A.5), (A.6), (A.8) (or (A.9)), $q_2 \geq 0$, and the nonrenewability constraint $q_2 \leq X_2$. This can be treated as an optimal control problem, with $q_2(\theta_2)$ the control variable and $\Pi_2(\theta_2)$ the state variable. But a simple way to solve it in this particular case is to first transform the objective function into a function of the extraction rate alone, by direct substitution.

From the definition of $\Pi_2(\theta_2)$, we have $R_2(\theta_2) = p_2 q_2(\theta_2) - \theta_2 q_2(\theta_2) - (b/2)q_2(\theta_2)^2 - \Pi_2(\theta_2)$. Substituting into (A.10), we may therefore rewrite it

$$EW_2 = \int_{\theta^L}^{\theta^H} \left[p_2 q_2(\theta_2) - \theta_2 q_2(\theta_2) - \frac{b}{2} q_2(\theta_2)^2 \right] f(\theta_2) d\theta_2 \\ - (1 - \alpha) \int_{\theta^L}^{\theta^H} \Pi_2(\theta_2) f(\theta_2) d\theta_2.$$

This represents the government's second-period objective as the expected value of the sum of the royalty receipts and the producers' surplus (the first term), minus that part of the expected producers' surplus which carries no weight in its objective (the second term).

Now substituting for $\Pi_2(\theta_2)$ from (A.7) and using integration by parts, we verify that

$$\int_{\theta^L}^{\theta^H} \Pi_2(\theta_2) f(\theta_2) d\theta_2 = \int_{\theta^L}^{\theta^H} q_2(\theta_2) F(\theta_2) d\theta_2 + \Pi_2(\theta^H).$$

We can therefore rewrite the government's objective function as

$$(A.11) \quad EW_2 = \int_{\theta^L}^{\theta^H} \left[p_2 q_2(\theta_2) - \theta_2 q_2(\theta_2) - \frac{b}{2} q_2(\theta_2)^2 \right. \\ \left. - (1 - \alpha) q_2(\theta_2) h(\theta_2) \right] f(\theta_2) d\theta_2 \\ - (1 - \alpha) \Pi_2(\theta^H)$$

where

$$h(\theta_t) = \frac{F(\theta_t)}{f(\theta_t)}, \quad t = 1, 2.$$

A.2. Period 1. Consider now the problem of period 1. Let $\phi(\tilde{\theta}_1; \theta_1)$ now represent a firm's expected total discounted surplus if it reports $\tilde{\theta}_1$ when its true cost parameter in period 1 is θ_1 . Hence

$$(A.12) \quad \phi(\tilde{\theta}_1; \theta_1) = p_1 q_1(\tilde{\theta}_1) - C(q_1(\tilde{\theta}_1), \theta_1) - R_1(\tilde{\theta}_1) + \delta \Psi(X_1 - q_1(\tilde{\theta}_1)).$$

In order for the firm to respond truthfully to the government's proposal of $q_1(\tilde{\theta}_1)$ and $R_1(\tilde{\theta}_1)$, conditions (A.2) and (A.3) must again hold, but with $(\tilde{\theta}_2, \theta_2)$ replaced by $(\tilde{\theta}_1, \theta_1)$. From those conditions, we derive that first-period incentive compatibility

implies

$$(A.13) \quad \left[p_1 - \theta_1 - b q_1(\tilde{\theta}_1) - \delta \frac{d\Psi(X_1 - q_1(\tilde{\theta}_1))}{dX_2} \right] \frac{dq_1(\tilde{\theta}_1)}{d\theta_1} - \frac{dR_1(\tilde{\theta}_1)}{d\theta_1} = 0$$

for $\tilde{\theta}_1 = \theta_1$

and

$$(A.14) \quad \frac{dq_1}{d\theta_1} \leq 0 \quad \text{for } \tilde{\theta}_1 = \theta_1.$$

In addition, if we let $\hat{\Pi}(\theta_1) \equiv \phi(\theta_1; \theta_1)$, we must have, using once more the envelope theorem,

$$(A.15) \quad \frac{d\hat{\Pi}}{d\theta_1} = -q_1(\theta_1).$$

Finally, the participation constraints for period 1 are

$$(A.16) \quad \hat{\Pi}(\theta_1) \geq 0 \quad \forall \theta_1 \in [\theta^L, \theta^H]$$

which, since $\hat{\Pi}(\theta_1)$ is a nonincreasing function of θ_1 because of (A.15), may again be replaced by the single constraint

$$(A.17) \quad \hat{\Pi}(\theta^H) \geq 0.$$

Notice that $\hat{\Pi}(\theta_1)$ is the expected total discounted producer surplus and not the period 1 producer surplus. A negative period 1 surplus would not inhibit participation by the firm provided expected total surplus is nonnegative.

As in the problem of period 2, these conditions are sufficient for global incentive compatibility.

The government's problem in period 1 is to choose $\{(R_1(\theta_1), q_1(\theta_1)) | \theta_1 \in [\theta^L, \theta^H]\}$ in order to maximize

$$(A.18) \quad V = \int_{\theta^L}^{\theta^H} \left\{ R_1(\theta_1) + \alpha [\hat{\Pi}(\theta_1) - \delta \Psi(X_1 - q_1(\theta_1))] \right. \\ \left. + \delta [\Gamma^a(X_1 - q_1(\theta_1)) + \alpha \Psi(X_1 - q_1(\theta_1))] \right\} f(\theta_1) d\theta_1$$

subject to the constraints (A.13) to (A.16) and to $0 \leq q_1(\theta_1) \leq X_1$.

We can apply the same approach as for period 2 in order to reduce the problem to that of choosing $q_1(\theta_1)$ and $\hat{\Pi}(\theta^H)$ to maximize

$$(A.19) \quad V = \int_{\theta^L}^{\theta^H} \left\{ p_1 q_1(\theta_1) - \theta_1 q_1(\theta_1) - \frac{b}{2} q_1(\theta_1)^2 - (1 - \alpha) q_1(\theta_1) h(\theta_1) \right. \\ \left. + \delta [\Psi(X_1 - q_1(\theta_1)) + \Gamma^a(X_1 - q_1(\theta_1))] \right\} f(\theta_1) d\theta_1 - (1 - \alpha) \hat{\Pi}(\theta^H)$$

subject to $0 \leq q_1(\theta_1) \leq X_1$ and to $\hat{\Pi}(\theta^H) \geq 0$.

B. *Generalization to an Arbitrary Number of Periods.* We show here how the method of solution employed in Section 4 for the case of two periods can be generalized to determine the closed-loop solution for an arbitrary number of periods. We retain the same notation principle. However some variables and functions which did not need to be explicitly dated, given the two-period context, now have to be.

Let T^a denote the optimal finite terminal date under asymmetry of information. It will depend on the sequence $\{\theta_t | t = 1, 2, \dots, T^a\}$ of realised θ_t 's. We assume that the price sequence $\{p_1, p_2, \dots\}$ is such that T^a exists for any such sequence.

Suppose T^a is known for the moment. The analysis of Section 4 then applies to periods T^a and $T^a - 1$, so that $q_{T^a}^a(\theta_{T^a}, X_{T^a})$, $\Pi_{T^a}(\theta_{T^a}, X_{T^a})$, $\Psi_{T^a}(X_{T^a})$, $\Gamma_{T^a}^a(X_{T^a})$ —the firm's output, its surplus, its expected surplus and the government's expected royalty in T^a under the incentive scheme—are given respectively by (24), (A.7), (26) and (27) where the subscript 2 has been replaced everywhere by the subscript T^a .

Now let $\hat{\Pi}_t(\theta_t, X_t^a)$ denote the expected total discounted surplus of the firm at period t . Hence

$$\hat{\Pi}_t(\theta_t, X_t^a) \equiv \Pi_t(\theta_t, X_t^a) + \delta \Psi_{t+1}(X_{t+1}^a),$$

where

$$(A.20) \quad X_t^a = X_{t-1}^a - q_{t-1}^a(\theta_{t-1}, X_{t-1}^a)$$

and where, corresponding to (26), we have

$$(A.21) \quad \Psi_t(X_t^a) \equiv \int_{\theta_L}^{\theta_H} \hat{\Pi}_t(\theta_t, X_t^a) f(\theta_t) d\theta_t.$$

We also have, corresponding to (27), that the expected sum of discounted royalties from period t to period T^a is given by

$$(A.22) \quad \Gamma_t^a(X_t^a) = \int_{\theta_L}^{\theta_H} \left\{ p_t q_t^a(\theta_t, X_t^a) - \theta_t q_t^a(\theta_t, X_t^a) - \frac{b}{2} q_t^a(\theta_t, X_t^a)^2 \right. \\ \left. + \delta [\Gamma_{t+1}^a(X_{t+1}^a) + \Psi_{t+1}(X_{t+1}^a)] \right\} f(\theta_t) d\theta_t - \Psi_t(X_t^a).$$

Then, by backward induction, the incentive compatibility and participation constraints in period t can be written exactly as in (A.13) to (A.17) provided we substitute the subscript t for the subscript 1 and replace $d\Psi/dX_2$ by $d\Psi_{t+1}/dX_{t+1}^a$ and $\hat{\Pi}$ by $\hat{\Pi}_t$, respectively.

In order to characterise the solution at any t , we repeat the substitution procedure leading to (A.19). We thus verify that the problem in period t is that of

choosing $q(\theta_t, X_t^a)$ and $\hat{\Pi}(\theta^H, X_t^a)$ to maximize

$$(A.23) \quad V_t = \int_{\theta^L}^{\theta^H} \left\{ p_t q_t(\theta_t, X_t^a) - \theta_t q_t(\theta_t, X_t^a) - \frac{b}{2} q_t(\theta_t, X_t^a)^2 \right. \\ \left. - (1 - \alpha) q_t(\theta_t, X_t^a) h(\theta_t) + \delta [\Psi_{t+1}(X_{t+1}^a) + \Gamma_{t+1}(X_{t+1}^a)] \right\} f(\theta_t) d\theta_t \\ - (1 - \alpha) \hat{\Pi}(\theta_t, X_t^a)$$

subject to the incentive compatibility and participation constraints and to $q_t \in [0, X_t^a]$, with X_t^a given by (A.20).

For an interior solution, $q_t^a(\theta_t, X_t^a)$ must satisfy

$$(A.24) \quad p_t - \theta_t - b q_t - (1 - \alpha) h(\theta_t) = \delta \left[\frac{d\Gamma_{t+1}^a(X_{t+1}^a)}{dX_{t+1}^a} + \frac{d\Psi_{t+1}(X_{t+1}^a)}{dX_{t+1}^a} \right].$$

This condition says that the current marginal surplus must be equal to the discounted value of expected marginal surpluses under the optimal contingent program for the remaining periods.

As in the two-period analysis, the solution for period t may however not be interior. To take into account the constraint $q_t \in [0, X_t^a]$ in writing the solution, let z_t^a denote the solution to

$$(A.25) \quad p_t - z_t^a - b X_t^a - (1 - \alpha) g(z_t^a) = \delta \left[\frac{d\Gamma_{t+1}^a(0)}{dX_{t+1}^a} + \frac{d\Psi_{t+1}(0)}{dX_{t+1}^a} \right]$$

and y_t^a the solution to

$$(A.26) \quad p_t - y_t^a - (1 - \alpha) g(y_t^a) = \delta \left[\frac{d\Gamma_{t+1}^a(X_t^a)}{dX_{t+1}^a} + \frac{d\Psi_{t+1}(X_t^a)}{dX_{t+1}^a} \right].$$

The complete solution can then be written

$$(A.27) \quad q_t^a(\theta_t, X_t^a) = \begin{cases} X_t^a & \text{if } \theta^L \leq \theta_t \leq \underline{\theta}_t^a \\ \tilde{q}_t^a(\theta_t, X_t^a) & \text{if } \underline{\theta}_t^a < \theta_t < \bar{\theta}_t^a \\ 0 & \text{if } \bar{\theta}_t^a \leq \theta_t \leq \theta^H \end{cases}$$

where $\tilde{q}_t^a(\theta_t, X_t^a)$ denotes the solution to (A.24) and where $\underline{\theta}_t^a = \min[z_t^a, \theta^H]$ and $\bar{\theta}_t^a = \max[y_t^a, \theta^L]$. Finding the solution for period t in fact involves finding \tilde{q}_t^a , $\underline{\theta}_t^a$ and $\bar{\theta}_t^a$. Note that $\underline{\theta}_t^a$ and $\bar{\theta}_t^a$ are implicit functions of X_t^a , which is endogenous for $t > 1$.

From (A.22) and (A.24) to (A.27), we may also verify that

$$\begin{aligned}
 (A.28) \quad & \frac{d\Gamma_t^a(X_t^a)}{dX_t^a} + \frac{d\Psi_t(X_t^a)}{dX_t^a} \\
 &= \int_{\theta_t^L}^{\theta_t^o} [p_t - \theta_t - bX_t^a] f(\theta_t) d\theta_t \\
 &\quad + \int_{\bar{\theta}_t^a}^{\bar{\theta}_t^o} [p_t - \theta_t - b\bar{q}_t^a(\theta_t, X_t^a)] f(\theta_t) d\theta_t \\
 &\quad + \delta \left[\frac{d\Gamma_{t+1}^a(X_{t+1}^a)}{dX_{t+1}^a} + \frac{d\Psi_{t+1}(X_{t+1}^a)}{dX_{t+1}^a} \right] [1 - F(\bar{\theta}_t^a)].
 \end{aligned}$$

Consider now the optimal terminal date. By definition, if T^a is the optimal finite terminal date for some sequence $\{\theta_t | t = 1, 2, \dots, T^a\}$, it must satisfy simultaneously

$$(A.29) \quad q_{T^a}^a(\theta_{T^a}, X_{T^a}^a) > 0$$

and

$$(A.30) \quad q_t^a(\theta_t, X_t^a) = 0 \quad \text{for all } t > T^a \quad \text{and for all } \theta_t \in [\theta^L, \theta^H].$$

Condition (A.29) of course requires $X_{T^a}^a > 0$, which itself holds if and only if the given sequence satisfies

$$(A.31) \quad \theta_t > \underline{\theta}_t^a \quad \text{for all } t < T^a.$$

This follows from (A.27), as does the fact that condition (A.29) also requires

$$(A.32) \quad \theta_{T^a} < \bar{\theta}_{T^a}^a.$$

Condition (A.30) will be satisfied if and only if either total exhaustion is desirable at T^a , i.e., $q_{T^a}^a(\theta_{T^a}, X_{T^a}^a) = X_{T^a}^a$, or positive extraction is never desirable for any firm type after T^a whatever the stock left over. In other words, if and only if either

$$(A.33) \quad \theta_{T^a} \leq \underline{\theta}_{T^a}^a$$

or

$$(A.34) \quad \theta_t^L \geq y_t^a \quad \text{for all } t > T^a \Leftrightarrow \bar{\theta}_t^a = \theta^L \quad \text{for all } t > T^a.$$

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